Theorem 1. Let A be $n \times m$ matrix over $\mathbb{R}$ and let h be a vector. Then the solution to the problem

$$\min_h ||Ah||_2 \qquad \text{such that} \qquad ||h||_2 = 1,$$

is the right singular vector corresponding to the least singular value in the singular value decomposition of the matrix A.

Proof. Let $A = U\Sigma V^T$, where $U \in R^{n\times n}$, $\Sigma \in R^{n\times m}$ and $V \in R^{m\times m}$ be the singular value decomposition of A. $\sigma_1 \geq \sigma_2 \geq, \ldots, \geq \sigma_m$ be the singular values of A ordered in decreasing manner in Σ.

$$\begin{aligned}\min_h ||Ah||_2 &= \min_h ||U\Sigma V^T h||_2 \\ &= \min_h ||\Sigma V^T h||_2 \qquad \because ||Ux||_2 = ||x||_2 \text{if U is orthogonal}\end{aligned}$$

Let $V^T h = y$ ($y \in \mathbb{R}^{m\times 1}$) $\implies Vy = h$ and also $||h||_2 = ||Vy||_2 = ||y||_2$. Therefore minimizing over h is same as minimizing over y.

$$\min_h ||Ah||_2 = \min_y ||\Sigma y||_2$$

Now, $\Sigma y = (\sigma_1 y_1, \sigma_2 y_2, \ldots, \sigma_m y_m, 0, \ldots, 0)$.
Now,

$$\begin{aligned}||\Sigma y||_2^2 &= \sum_{i=1}^{n} |\sigma_i y_i|^2 \\ &= \sum_{i=1}^{m} |\sigma_i y_i|^2 \\ &\geq \sigma_m^2 \sum_{i=1}^{m} |y_i|^2 \\ &\geq \sigma_m^2 \qquad ||y||_2 = 1\end{aligned}$$

The above minimum is achieved for $y = (0, 0, \ldots, 1)$ (all zeros except 1 at m^{th} location).

$\therefore min_y \ ||\Sigma y||_2 = \sigma_m$ for $y = (0, 0, \ldots, 1)$ where 1 is at m^{th} location.

Since $h = Vy$, the solution h is the m^{th} column (last column) of orthogonal matrix V i.e, solution h is the right singular vector corresponding to the least singular value.

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