Visual-Inertial-Aided Navigation for High-Dynamic Motion in Built Environments Without Initial Conditions

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1 Motivation

Paper: [1]. The motivation of the approach lies in the fact that special initialization procedure takes too long and cannot be used in the applications such as human-mounted localization system for first responders. Second, inertial observations which are observed in the body frame of reference can be integrated in an arbitray frame of reference.

2 Brief

The paper proposes an approach to visual inertial 6-DoF state estimation using EKF. Highlights include:

- Unlike exisiting work where input (acceleration and gyroscope data) is transformed into the navigation world frame and integration carried out there, this method carries out integration in the body frame at the start of integration interval and then these are transformed into the navigation frame. The implication being, no specification of initial conditions (roll α and pitch β angles) until the preintegrated factors are generated.
- This reparameterization removes uncertainity over initial orientation (the initial pose of the robot is the world navigation frame which is known exactly) which has non-linear effect on the propagation of state (which will be clear if the jacobians are evaluated). But, due to this reparameterization uncertainity creeps in over gravity vector in the navigation frame.
- No special initialization procedure is necessary
- Introduces a new approach to fuse inertial terms by introducing preintegration theory.
- Preintegrated terms between t_i and t_j generate constraints that can either be used during prediction step of EKF or as observations during update step.
- Initial velocity and gravity vector are unobserved due to this formulation but are recovered in a linear fashion in subsequent steps. This is accompomlished using different sensor suite. Here, stereo camera is used.
- Most importantly, as a result of preintegration, a number of inertial measurements can be summed up in one constraint, an idea which was further developed in [2] in optimization framework to render the problem to be solved in real-time using incremental solvers such iSAM2 [3].

3 Assumptions

- To recover unobservable gravity vector and initial velocity, a different sensor suite has to be used for their estimation.

4 Method

IMU model is given by,

$$
\begin{aligned}
{}_{\mathrm{B}}\tilde{\boldsymbol{\omega}}_{\mathrm{WB}}(t) &= {}_{\mathrm{B}}\boldsymbol{\omega}_{\mathrm{WB}}(t) + \mathbf{b}^g(t) + \boldsymbol{\eta}^g(t) \qquad (1)\\
{}_{\mathrm{B}}\tilde{\mathbf{a}}(t) &= \mathtt{R}_{\mathrm{WB}}^{\mathsf{T}}(t)\left({}_{\mathrm{W}}\mathbf{a}(t) - {}_{\mathrm{W}}\mathbf{g}\right) + \mathbf{b}^a(t) + \boldsymbol{\eta}^a(t), \qquad (2)
\end{aligned}
$$

Kinematics can be stated as:

$$
\dot{\boldsymbol{\theta}} = E_{\mathrm{WB}}\ {}_{\mathrm{B}}\boldsymbol{\omega}_{\mathrm{WB}}, \qquad {}_{\mathrm{W}}\dot{\mathbf{v}} = {}_{\mathrm{W}}\mathbf{a}, \qquad {}_{\mathrm{W}}\dot{\mathbf{p}} = {}_{\mathrm{W}}\mathbf{v}, \qquad (3)
$$

where $\boldsymbol{\theta} = \begin{pmatrix}\alpha\ \beta\ \gamma\end{pmatrix}^{\mathsf{T}}$ is Euler angles and E is given by (α and β being roll and pitch angles):

$$
E_{\mathrm{WB}} = \begin{bmatrix} 1 & \sin\alpha\tan\beta & \cos\alpha\tan\beta \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha\sec\beta \end{bmatrix}
$$

Note in the above that as $\beta \to \pm 90°$, the matrix become singular. The state at time $t + \Delta t$ is obtained by integrating Eq. (3):

$$\begin{aligned}
\boldsymbol{\theta}(t+\Delta t) &= \boldsymbol{\theta}(t) + \int_t^{t+\Delta t} E_{\mathrm{WB}}\ {}_{\mathrm{B}}\boldsymbol{\omega}_{\mathrm{WB}}(\tau) d\tau \\
{}_{\mathrm{W}}\mathbf{v}(t+\Delta t) &= {}_{\mathrm{W}}\mathbf{v}(t) + \int_t^{t+\Delta t} {}_{\mathrm{W}}\mathbf{a}(\tau) d\tau \\
{}_{\mathrm{W}}\mathbf{p}(t+\Delta t) &= {}_{\mathrm{W}}\mathbf{p}(t) + \int_t^{t+\Delta t} {}_{\mathrm{W}}\mathbf{v}(\tau) d\tau + \iint_t^{t+\Delta t} {}_{\mathrm{W}}\mathbf{a}(\tau) d\tau^2. \qquad (4)
\end{aligned}$$

The integrals in the above can be evaluated using two approaches. The standard approach has been to transform the raw sensor data into navigation frame and then integrate 4.1 and preintegration 4.2.

4.1 Inertial Integration

Typically ${}_{\mathrm{W}}\mathbf{a}$ and ${}_{\mathrm{B}}\boldsymbol{\omega}_{\mathrm{WB}}$ are transformed to the navigation frame (here W frame) and integration is carried out there. Assuming that ${}_{\mathrm{W}}\mathbf{a}$ and ${}_{\mathrm{B}}\boldsymbol{\omega}_{\mathrm{WB}}$ remain constant in the time interval $[t, t+\Delta t]$, we can write:

$$\begin{aligned}
\boldsymbol{\theta}_{\mathrm{WB}}(t+\Delta t) &= \boldsymbol{\theta}_{\mathrm{WB}}(t) + \ E_{\mathrm{WB}}\ {}_{\mathrm{B}}\boldsymbol{\omega}_{\mathrm{WB}}(t)\Delta t \\
{}_{\mathrm{W}}\mathbf{v}(t+\Delta t) &= {}_{\mathrm{W}}\mathbf{v}(t) + {}_{\mathrm{W}}\mathbf{a}(t)\Delta t \\
{}_{\mathrm{W}}\mathbf{p}(t+\Delta t) &= {}_{\mathrm{W}}\mathbf{p}(t) + {}_{\mathrm{W}}\mathbf{v}(t)\Delta t + \frac{1}{2}{}_{\mathrm{W}}\mathbf{a}(t)\Delta t^2. \qquad (5)
\end{aligned}$$

Using Eqs. (1)–(2), we can write ${}_{\mathrm{W}}\mathbf{a}$ and ${}_{\mathrm{B}}\boldsymbol{\omega}_{\mathrm{WB}}$ as a function of the IMU measurements, hence (5) becomes

$$\begin{aligned}
\boldsymbol{\theta}(t+\Delta t) =&\ \boldsymbol{\theta}(t) + E(\boldsymbol{\theta})\left(\left(\tilde{\boldsymbol{\omega}}(t) - \mathbf{b}^g(t) - \boldsymbol{\eta}^{gd}(t)\right)\Delta t\right) \\
\mathbf{v}(t+\Delta t) =&\ \mathbf{v}(t) + \mathbf{g}\Delta t + \mathtt{R}(t)\left(\tilde{\mathbf{a}}(t) - \mathbf{b}^a(t) - \boldsymbol{\eta}^{ad}(t)\right)\Delta t \\
\mathbf{p}(t+\Delta t) =&\ \mathbf{p}(t) + \mathbf{v}(t)\Delta t + \frac{1}{2}\mathbf{g}\Delta t^2 + \frac{1}{2}\mathtt{R}(t)\left(\tilde{\mathbf{a}}(t) - \mathbf{b}^a(t) - \boldsymbol{\eta}^{ad}(t)\right)\Delta t^2, \qquad (6)
\end{aligned}$$

Eqs. (6) give the standard propagation of state from t to $t + \Delta t$ in an EKF.

4.2 Inertial Preintegration

Re-writing Eqs. (4) by integrating from t_i to t_j, we have,

$$\begin{aligned}
\boldsymbol{\theta}_j &= \boldsymbol{\theta}_i + \int_{t_i}^{t_j} E_{\mathrm{W}\tau}(\boldsymbol{\theta}(\tau))\ {}_{\mathrm{B}}\boldsymbol{\omega}_{\mathrm{WB}}(\tau) d\tau \\
{}_{\mathrm{W}}\mathbf{v}_j &= {}_{\mathrm{W}}\mathbf{v}_i + \int_{t_i}^{t_j} {}_{\mathrm{W}}\mathbf{a}(\tau) d\tau \\
{}_{\mathrm{W}}\mathbf{p}_j &= {}_{\mathrm{W}}\mathbf{p}_i + \int_{t_i}^{t_j} {}_{\mathrm{W}}\mathbf{v}(\tau) d\tau + \iint_t^{t_j} {}_{\mathrm{W}}\mathbf{a}(\tau) d\tau^2. \qquad (7)
\end{aligned}$$

The above can be modified as follows (dropping subscripts and separating gravity out) and using the fact that $\mathtt{R}_{\mathrm{W}t} = \mathtt{R}_{\mathrm{W}t_i}\mathtt{R}_{t_i t}$,

$$\begin{aligned}
\boldsymbol{\theta}_j &= \boldsymbol{\theta}_i + E_{\mathrm{W}t_i}\int_{t_i}^{t_j} E_{t_i t}\ \left(\left(\tilde{\boldsymbol{\omega}}(t) - \mathbf{b}^g(t) - \boldsymbol{\eta}^{gd}(t)\right)\right) dt \\
\mathbf{v}_j &= \mathbf{v}_i + \int_{t_i}^{t_j} \mathbf{g} dt + \mathtt{R}_{\mathrm{W}t_i}\int_{t_i}^{t_j} \mathtt{R}_{t_i t}(t)\left(\tilde{\mathbf{a}}(t) - \mathbf{b}^a(t) - \boldsymbol{\eta}^{ad}(t)\right) dt \\
\mathbf{p}_j &= \mathbf{p}_i + \iint_{t_i}^{t_j} \mathbf{g} dt + \int_{t_i}^{t_j} \mathbf{v}(t) dt + \mathtt{R}_{\mathrm{W}t_i}\iint_t^{t_j} \mathtt{R}_{t_i t}\left(\tilde{\mathbf{a}}(t) - \mathbf{b}^a(t) - \boldsymbol{\eta}^{ad}(t)\right) dt^2. \qquad (8)
\end{aligned}$$

Separating out the integral terms as,

$$\begin{aligned}
\Delta\boldsymbol{\theta}_{ij} &= \int_{t_i}^{t_j} E_{t_i t}\ \left(\left(\tilde{\boldsymbol{\omega}}(t) - \mathbf{b}^g(t) - \boldsymbol{\eta}^{gd}(t)\right)\right) dt \\
\Delta\mathbf{v}_{ij} &= \int_{t_i}^{t_j} \mathtt{R}_{t_i t}(t)\left(\tilde{\mathbf{a}}(t) - \mathbf{b}^a(t) - \boldsymbol{\eta}^{ad}(t)\right) dt \\
\Delta\mathbf{p}_{ij} &= \iint_t^{t_j} \mathtt{R}_{t_i t}\left(\tilde{\mathbf{a}}(t) - \mathbf{b}^a(t) - \boldsymbol{\eta}^{ad}(t)\right) dt^2. \qquad (9)
\end{aligned}$$

Substituting back (9) into (8), we get,

$$\boldsymbol{\theta}_j = \boldsymbol{\theta}_i + E_{\mathrm{w}t_i} \Delta\boldsymbol{\theta}_{ij}$$
$$\mathbf{v}_j = \mathbf{v}_i + \int_{t_i}^{t_j} \mathbf{g} dt + \mathtt{R}_{\mathrm{w}t_i} \Delta\mathbf{v}_{ij}$$
$$\mathbf{p}_j = \mathbf{p}_i + \iint_{t_i}^{t_j} \mathbf{g} dt + \int_{t_i}^{t_j} \mathbf{v}(t) dt + \mathtt{R}_{\mathrm{w}t_i} \Delta\mathbf{p}_{ij}. \tag{10}$$

Entire crux of preintegration lies in the simple manipulation of (7) giving rise to preintegration terms in (9). Integration in (9) can be performed without any knowledge of initial conditions! While $\Delta\boldsymbol{\theta}_{ij}, \Delta\mathbf{v}_{ij}$ refer to actual change in the states between t_i and t_j, $\Delta\mathbf{p}_{ij}$ does not since it depends on velocity in the navigation frame at the start of integration interval.

4.3 Initial condition recovery

Once the preintegrated measurements are calculated as in (9), to recover state $\boldsymbol{\theta}_j, \mathbf{v}_j, \mathbf{p}_j$ in (10) one still needs an estimate of gravity ${}_{\mathrm{w}}\mathbf{g}, {}_{\mathrm{w}}\mathbf{v}_i$. Note that the world frame here is the initial pose of the robot.

Since initial position and attitude is fixed, only gravity and initial velocity need to be estimated. Initial velocity becomes observable as follows,

$$\int_{t_1}^{t_2} \mathbf{v}(t) dt = \mathbf{p}_2 - \mathbf{p}_1 - \iint_{t_1}^{t_2} \mathbf{g} dt - \mathtt{R}_{\mathrm{w}t_1} \Delta\mathbf{p}_{12}. \tag{11}$$
$$\mathbf{v}_1 \Delta t_{12} = \mathbf{p}_2 - \mathbf{p}_1 - \mathbf{g}\Delta t_{12} - \mathtt{R}_{\mathrm{w}t_1} \Delta\mathbf{p}_{12}. \tag{12}$$
$$\mathbf{v}_1 = \frac{\mathbf{p}_2 - \mathbf{p}_1 - \mathbf{g}\Delta t_{12} - \mathtt{R}_{\mathrm{w}t_1} \Delta\mathbf{p}_{12}}{\Delta t_{12}}. \tag{13}$$

Hence, given two positional estimates $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{g}$, $\mathbf{v}_1$ can be recovered. To recover gravity, one more positional observations are needed.

$$\mathbf{p}_2 = \mathbf{p}_1 + \mathbf{g}\Delta t_{12}^2 + \mathbf{v}_1 \Delta t_{12} + \mathtt{R}_{\mathrm{w}t_1} \Delta\mathbf{p}_{12}. \tag{14}$$
$$\mathbf{p}_3 = \mathbf{p}_2 + \mathbf{g}\Delta t_{23}^2 + \mathbf{v}_2 \Delta t_{12} + \mathtt{R}_{\mathrm{w}t_2} \Delta\mathbf{p}_{23}. \tag{15}$$
$$\mathbf{v}_2 = \mathbf{v}_1 + \mathbf{g}\Delta t_{12} + \mathtt{R}_{\mathrm{w}t_i} \Delta\mathbf{v}_{ij} \tag{16}$$

Substituting (16) into (15),

$$\mathbf{p}_3 = \mathbf{p}_2 + \mathbf{g}\Delta t_{23}^2 + \{\mathbf{v}_1 + \mathbf{g}\Delta t_{12} + \mathtt{R}_{\mathrm{w}t_i} \Delta\mathbf{v}_{ij}\} \Delta t_{12} + \mathtt{R}_{\mathrm{w}t_2} \Delta\mathbf{p}_{23}. \tag{17}$$

And substituting $\mathbf{v}_1$ from (13) into above gravity can be estimated. If one were to use cameras, these positional estimates can be obtained.

4.4 Bias compensation

Since delta measurements in (9) depend on bias, and biases being part of the state to be estimated, they could change in an optimization based framework. Instead of recomputing them from scratch at the new bias estimate, the paper proposes to use first order approximation of measurements given a small update using Taylors expansion.

5 Conclusions

1. Integration is performed in the body frame at the start of integration rather than the global navigation frame.
2. Introduction of preintegration overcomes the following two limitations
   - IMU frequency can be as high as 1KHz, summarizing multiple measurements into one single constraint and thus making updates computationally feasible in a framework such as EKF.
   - In case of smoothing based frameworks (not covered in this work), states would have to be added at the frequency of IMU but this can again be overcome by generating preintegration terms. This idea has been explored in great detail at [2].
3. By not relying on special initialization procedure for attitude, gravity and initial velocity can be estimated in linear fashion and most importantly, uncertainty over initial attitude which is shown to have a non-linear effect on the state estimation (due to the rotation matrices) has been circumvented at the cost of introducing uncertainity over gravity. The caveat being, gravity can be estimated linearly wrt the estimated states and makes estimation to be solved using linear solvers as opposed to non-linear methods for state estimation.

References

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